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Marginal Loss Calculation in Competitive Electrical Energy Markets

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Abstract— **The core of FERC proposed Standard Market Design (SMD) in the US is Locational Marginal Pricing (LMP) for electrical energy by which the energy prices are to be determined based on marginal costs in order to promote economic efficiency. One of the main challenging issues in implementing LMP methodology is the pricing of marginal losses, which requires accurate analysis of transmission losses and incorporating the effects of marginal losses into the optimal generation scheduling process. This paper presents two optimal generation scheduling algorithms where the effects of marginal losses are reflected through the use of penalty factors and delivery factors. Both algorithms are consistent with the overall SMD energy pricing approach, allowing for the calculation of three different LMP components: marginal energy cost, marginal congestion cost, and marginal loss cost. The algorithms are benchmarked with the NYISO method on a simple two-bus system, and then examined using a more realistic five-bus system. Some important issues on the implementation of marginal loss pricing in competitive energy markets are discussed.**

Index Terms—**Locational marginal price (LMP), Marginal loss calculation, Penalty factors, Delivery factors, Generation resource scheduling, Spot energy market.**

I. INTRODUCTION

Transmission losses are always involved as moving power from generation resources to loads because of the resistance of each element in the transmission system. These losses appear as additional electrical load, requiring the generators to produce additional power to compensate the losses. For a typical transmission system, the annual loss factor is in the range of 2~5% of total energy consumption. However, losses vary greatly as a function of network configuration, generator locations and outputs, and customer locations and demands. In particular, losses during heavy loading period are often much higher than under average loading condition because a quadratic relationship between losses and line flows can be assumed for most transmission devices.

Transmission losses are essentially important in determining the optimal scheduling of generation resources. The generation scheduling process involves two fundamental tasks: (1) optimal combination of generation resources that satisfies system load and required operating reserves subject to operational constraints; (2) optimal utilization of generation resources such that the total system energy production costs is minimized. In the traditional optimal generation scheduling programs, the effects of incremental transmission losses are usually considered through the use of penalty factors associated with individual generation facilities [1]. These penalty factors are used to obtain the equivalent generation production costs. Thus, the marginal losses are included in the total cost minimization of system operation.

The introduction of competitive electricity markets has added the complexity to the optimal generation scheduling problems incorporating marginal transmission losses. The core of FERC proposed Standard Market Design (SMD) is Locational Based Marginal Pricing (LMP or LBMP) for electrical energy by which the energy prices and the associated transmission usage chargers are to be determined based on marginal costs in order to promote economic efficiency [2]. One of the main challenging issues in implementing LMP methodology is the pricing of marginal transmission losses, which requires accurate analysis of transmission losses and incorporating the effects of marginal losses into the optimal generation scheduling programs.

This paper presents two optimal generation scheduling algorithms where the effects of marginal losses are reflected through the use of penalty factors and delivery factors. Both algorithms are consistent with the overall SMD energy pricing approach, allowing for the calculation of three different LMP components: marginal energy cost, marginal congestion cost, and marginal loss cost. The proposed algorithms are benchmarked with the NYISO method on a simple two-bus system, and then examined using a more realistic five-bus system. Some important issues on marginal loss pricing in competitive energy markets are discussed.

II. POPOSED TWO ALGORITHMS

This section describes the proposed algorithms for optimal generation scheduling and LMP calculation involving marginal transmission losses. The proposed algorithms assume a DC load flow model for transmission network, which is a simplified version of accurate AC load

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flow model. The DC modeling approach has been widely accepted in the electric power industry as practical means to incorporate transmission constraints into the optimal generation scheduling programs.

A. List of Symbols

- NB: number of buses
- NG: number of generators
- ND: number of loads
- NK: number of branches
- C_g: energy bid price of generator g ($/MWh)
- G_g: output of generator g (MW)
- D_d: demand level of load d (MW)
- P_B: net injection at bus B (MW)
- $Limit_k$: capacity rating of branch k (MW)
- F_k: actual flow on branch k (MW)
- R_k: resistance of branch k
- $GSF_{k\text{-}B}$: Generation Shift Factor representing the sensitivity of the flow on branch k to a change of net injection at bus B. GSFs can be directly calculated from the network bus impedance matrix [1].
- PF_g, PF_B: penalty factors associated with generator g and bus B, respectively
- DF_g, DF_d: delivery factors associated with generator g and load d, respectively
- SMP: system marginal energy price (i.e. the shadow price of system load constraint
- MCC_B: LMP congestion component representing the marginal cost of congestion at bus B relative to the reference bus
- MLC_B: LMP component representing the marginal cost of losses at bus B relative to the reference bus
- SP_k: shadow price of network constraint k

B. Penalty Factors and Delivery Factors

The Penalty Factor associated with any bus on the transmission system is defined as the increase required in injection at that bus to supply an increase in withdrawn at the system reference bus with all other bus net injections held constant [1,3]. Mathematically, the Penalty Factor for bus B can be calculated as:

$$PF_B = 1/(1 - \frac{\partial P_{Loss}}{\partial P_B}) \tag{1}$$

where $\frac{\partial P_{Loss}}{\partial P_B}$ represents the incremental transmission losses and can be calculated by

$$\frac{\partial P_{Loss}}{\partial P_B} = \frac{\partial}{\partial P_B}(\sum_{k=1}^{NK} F_k^2 \times R_k) \tag{2}$$

Equation (2) can then be reformulated as Equation (3) and Equation (4).

$$\frac{\partial P_{Loss}}{\partial P_B} = \frac{\partial}{\partial P_B}(\sum_{k=1}^{NK}\left(\sum_{B=1}^{NB} GSF_{k-B} \times P_B)\right)^2 \times R_k) \tag{3}$$

$$\frac{\partial P_{Loss}}{\partial P_B} = 2 \times \sum_{k=1}^{NK} GSF_{k-B} \times \left(\sum_{B=1}^{NB} GSF_{k-B} \times P_B\right) \times R_k \tag{4}$$

In the marginal loss pricing formulation, the Delivery Factors are also needed in addition to the Penalty Factors. The Delivery Factor for bus B is defined as follows:

$$DF_B = \frac{1}{PF_B} \tag{5}$$

C. Algorithm Based On Equivalent Energy Bid Prices – Algorithm 1

With this algorithm, the generator energy bid prices are multiplied by the Penalty Factors to account for incremental transmission losses in the scheduling process. That is, the marginal losses are included in the total cost minimization of system operation through equivalent energy bid prices. In the simplest version, the optimal generation scheduling problem with the consideration of marginal losses can be formulated as:

$$\text{Min} \quad \sum_{g=1}^{NG} PF_g \times c_g \times G_g \tag{6-1}$$

$$\text{s.t.} \quad \sum_{g=1}^{NG} G_g = \sum_{d=1}^{ND} D_d + Losses \tag{6-2}$$

$$\sum_{B=1}^{NB} GSF_{k-B} \times P_B \le Limit_k \tag{6-3}$$

After obtaining the optimal solution of generation scheduling, the LMP at any bus B can be calculated as the sum of the following three LMP components: system marginal energy cost, marginal congestion cost and marginal loss cost.

$$LMP_B = SMP + MCC_B + MLC_B \tag{6-4}$$

$$MCC_B = -(\sum_{k \in K} GSF_{k-B} \times SP_k \times DF_B) \tag{6-5}$$

$$MLC_B = SMP \times (DF_B - 1) \tag{6-6}$$

D. Algorithm Based On Net Bus Injections – Algorithm 2

With this algorithm, the generator outputs and load levels are multiplied by the Delivery Factors to account for incremental transmission losses in the optimal scheduling process. That is, the marginal losses are considered in the supply and demand constraint through net bus MW injections. The optimal generation scheduling problem considering marginal losses can be formulated as:

$$\text{Min} \sum_{g=1}^{NG} c_g \times G_g \qquad (7\text{-}1)$$

$$\text{s.t.} \quad \sum_{g=1}^{NG} DF_g \times G_g = \sum_{d=1}^{ND} DF_d \times D_d \qquad (7\text{-}2)$$

$$\sum_{B=1}^{NB} GSF_{k-B} \times P_B \le Limit_k \qquad (7\text{-}3)$$

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III. Benchmarking with NYISO Two-Bus Example

This section presents the benchmarking of the proposed algorithms against the NYISO LBMP calculation method, using a simple two-bus system provided in the NYISO Transmission & Dispatching Operations Manual [3]. The initial system operating condition and the assumptions used in this example are shown in Figure 1.

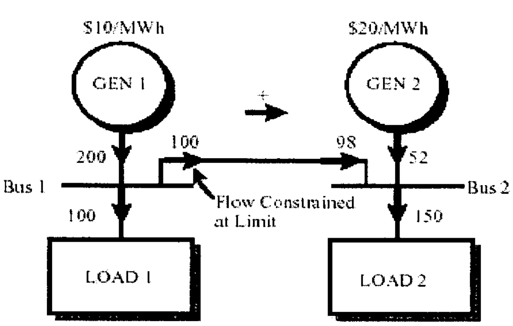

Figure 1: NYISO Two-Bus Example

With the given assumptions and initial system operating conditions, the system is re-dispatched using the proposed generation scheduling algorithms and LMP calculation methods. Table 1 shows the optimal solutions obtained from the proposed algorithms compared to the results of NYISO methodology.

Table 1: Benchmarking Results

Bus	Elements	NYISO	Alg. 1	Alg. 2
Bus 1	Dispached MW	200	200	200
	Load MW	100	100	100
	Penanly Factor	1.0204	1.0204	1.0204
	Delivery Factor	0.9800	0.9800	0.9800
	Gen Shift Factor	1.0000	1.0000	1.0000
	Constraint Cost $/MWh	9.6000	9.7959	9.6000
	Energy Component	20	20	20
	Loss Component	-0.4000	-0.4000	-0.4000
	Congestion Component	-9.6000	-9.6000	-9.6000
	LBMP	**10.00**	**10.00**	**10.00**
Bus 2	Dispached MW	52	52	50
	Load MW	150	150	150
	Penanly Factor	1.0000	1.0000	1.0000
	Delivery Factor	1.0000	1.0000	1.0000
	Gen Shift Factor	0	0	0
	Constraint Cost $/MWh			
	Energy Component	20	20	20
	Loss Component	0	0	0
	Congestion Component	0	0	0
	LBMP	**20.00**	**20.00**	**20.00**
Transmission Losses		2	2	2

It can be seen that the scheduled generation and LMP results are identical to the results of NYISO methodology. It has to be mentioned that the proposed algorithms may not fully represent the current NYISO methodology even though they are consistent with the overall SMD energy pricing approach. No detailed information has been released by the NYISO on the modeling of optimal generation scheduling process. However, the above benchmarking results do provide certain confidence on the proposed algorithms.

IV. Test Results on Five-Bus Example System

This section examines the proposed algorithms based on a more realistic five-bus system [4]. The system configuration is shown in Figure 2 representing a transmission constrained operating condition. The initial generation scheduling and LMP calculation results without considering transmission losses are given in Table 2, which is identical to the results presented in [4]. Table 3 shows the estimated losses and marginal loss factors (i.e., penalty factors and delivery factors) based on the load flow solution from the initial generation scheduling.

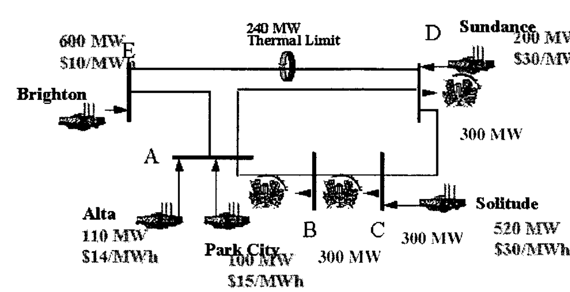

Figure 2: PJM-ISO Five-Bus Example

Table 2: Initial Scheduling and LMP (no losses)

Generator	Capacity MW	Bid Price $/MWh	Dispatched MW	LMP ($/MWh)
Alta	110	14	110	15
Park City	100	15	66	21.14
Solitude	520	30	0	23.51
Sundance	200	30	124	30
Brighton	600	10	600	10.44
Total			900	
Constraint Cost ($/MW)	40.7061			

Table 3: Estimated Losses and Loss Factors

Bus	Penalty Factor	Delivery Factor
A	1.0096	0.9905
B	0.9887	1.0114
C	0.9871	1.0131
D	1.0000	1.0000
E	1.0143	0.9859
Losses MW	8.765	

Table 4 shows the scheduled generation and LMP considering the effects of marginal losses. Additional generation is produced from the generator Sundance to compensate the losses. With Algorithm 1, exact amount of additional generation is scheduled to balance the power losses estimated based on the initial scheduling. The power losses estimated based on load flow solution is called average losses. However, excessive generation output, as much as twice of the average losses, is scheduled when using algorithm 2 where the power losses are implicitly expressed in the system supply and demand constraint. This observation is consistent with the concept that the marginal losses are twice the average losses [5].

Table 4: Scheduling and LMP (with losses)

Generator	*Algorithm 1*		*Algorithm 2*	
	Dispatched MW	LMP ($/MWh)	Dispatched MW	LMP ($/MWh)
Alta	110	15	110	15
Park City	66.002	21.4727	66.002	21.6568
Solitude	0	23.8771	0	24.0228
Sundance	132.754	30	141.5099	30
Brighton	600	10.4813	600	10.3929
Total	908.756		917.5119	
Constraint Cost ($/MW)	41.3135		39.9284	

Table 5 gives the LMP components calculated by the proposed algorithms. As with the initial scheduling, the system marginal energy cost is 30 $/MWh under the given system loading condition and transmission capability. It is also observed that the marginal loss costs are the same for the two algorithms, because the same set of delivery factors estimated based on the initial scheduling are used. In fact, more accurate optimal scheduling process may use iterative technique to update transmission losses and marginal loss factors and reschedule generation resources until certain convergence criteria satisfied [6]. Minor difference in congestion costs can be observed attributed to the difference of the two algorithms in the modeling of transmission constraints.

Table 5: LMP Components (with losses)

Bus	Energy ($/MWh)	Loss ($/MWh)		Congestion ($/MWh)	
		Alg. 1	Alg. 2	Alg. 1	Alg. 2
A	30	-0.2866	-0.2866	-14.7134	-14.7134
B	30	0.3433	0.3433	-8.8706	-8.6865
C	30	0.3929	0.3929	-6.5158	-6.3701
D	30	0	0	0	0
E	30	-0.4234	-0.4234	-19.0953	-19.1837

V. Disucssions and Conclusions

The marginal loss methodology is consistent with the LMP principles of competitive electrical energy markets. If the energy cost is computed on the basis of location and time, then it is logical to treat the loss calculation in the same manner. The marginal loss calculation method is also compatible with the approach for calculating congestion cost under the established spot electrical energy markets. Marginal loss calculation can provide customers a price signal reflecting the actual cost of losses associated with their usage of transmission facilities.

Both NYISO and ISO-NE have implemented the marginal loss pricing methodology in their Day-Ahead and Real-Time energy markets [6,7]. The loss component of LMP is calculated by the ISO's security constrained unit commitment and dispatch software and represents the cost of marginal losses, in $/MWh, at each location relative to the reference bus. Currently, PJM-ISO is investigating hardware/software and other necessary resources to account for marginal losses in the dispatch of energy and the calculation of LMP [8].

There are alternative ways to model the marginal losses in generation scheduling and billing calculation. In reality, the implementation of LMP principle involves both technical and regulatory issues. The two algorithms presented in this paper are consistent with the overall SMD energy pricing approach, considering the marginal cost of losses both when dispatching generators and calculating prices. However, the dispatch results and the calculated prices are not the same by the two algorithms due to the difference in the formulation of transmission-constrained generation scheduling and LMP calculation process.

Further studies are still needed to investigate how the bid-based priority of generators and the loading of transmission network can be appropriately adjusted in the solution process by the effects of marginal loss factors. In addition, the problem of over-estimated power losses by marginal loss calculation should also be properly addressed. It is also desirable to develop a reference bus independent methodology for calculating marginal loss factors to ensure the fairness of marginal loss pricing to all transmission customers [9,10].

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VII. BIOGRAPHIES

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